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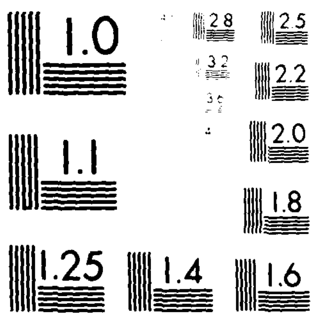

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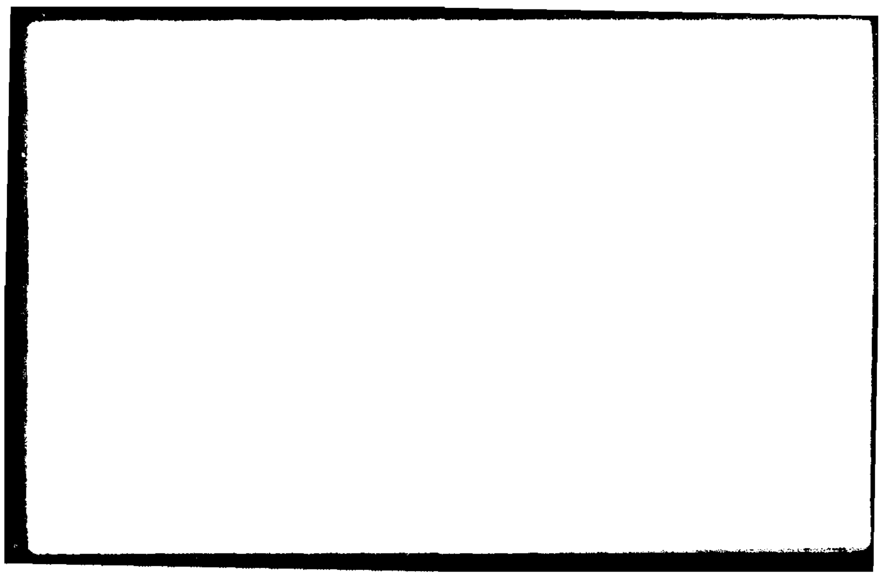

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ON THE BEHAVIOR OF PLATES LAMINATED OF BIMODULUS COMPOSITE MATERIALS

by

J. N. Reddy
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia, USA 24061
and
C. W. Bert
School of Aerospace, Mechanical and Nuclear Engineering
University of Oklahoma
Norman, Oklhaoma, USA 73019

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J. N. Reddy
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia, USA 24061

C. W. Bert
School of Aerospace, Mechanical and Nuclear Engineering
University of Oklahoma
Norman, Oklahoma, USA 73019

An exact and finite-element analysis is used to predict the static bending and natural vibration response of rectangular plates laminated of composite material having different elastic properties depending upon whether the fiber-direction strains are tensile or compressive. The analysis is based on a theory that accounts for anisotropy, bending-stretching coupling, thickness shear deformation, and both coupling inertia and rotatory inertia. The finite-element results are found to agree very closely with the exact closed-form solutions in the case of a cross-ply rectangular plate having simply supported edges and subjected to sinusoidally distributed normal-pressure loading or temperature loading.

1. Introduction

Certain fiber-reinforced materials, especially those with very soft matrix material (e.g., cord-rubber composites), exhibit different elastic behavior depending upon whether the fiber-direction strain is tensile or compressive [1-3]. In other words, the tangent modulus in tension is quite different from the tangent modulus in compression. As a first approximation the uniaxial stress-strain behavior of such materials is often represented as being linear with different slopes (i.e., elastic moduli) depending upon the sign of the fiber-direction strain. Such a

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material is called a bimodulus composite material. Several macroscopic material models appropriate for bimodulus fiber-reinforced composites have been proposed and are reviewed in [4]. It has been shown that the fiber-governed symmetric-compliance model proposed in [5] agrees well with experimental data for several materials with drastically different elastic properties in tension and compression.

Prior to the works of the present authors, the literature available in English on bending and vibration analyses of bimodulus plates was quite sparse and largely concerned with the bending of bimodulus isotropic-material plates. Shapiro [6] considered the simple problem of a circular plate under a pure bending moment at its edge. Kamiya [7] analyzed the large-deflection behavior of clamped circular plates using a finite-difference technique. In [8], Kamiya used the Galerkin method to analyze the large-deflection behavior of rectangular plates under sinusoidally distributed loading. The effect of thickness shear deformation was included in the simple case of cylindrical bending by Kamiya [9]. The only previous analyses applicable to anisotropic bimodulus material are the works of Jones and Morgan [10], who treated cylindrical bending of a thin, cross-ply laminate, and the closed-form solutions of Kincannon and Bert [11,12] for clamped elliptic plates. All of the previous thermoelastic analyses of bimodulus-material plates were also limited to isotropic bimodulus materials and midplane-symmetric temperature changes [13-17]. Apparently, there were no published papers on the vibration of bimodulus plates.

In the last two years, the present authors have contributed a series of papers to the literature on bimodulus rectangular plates [18-23]. These works are apparently more general than any analyses that have appeared in the open literature. More specifically, these works differ from previous works in the following respects:

1. The material of each layer is both elastically and thermoelastically orthotropic and bimodular.
2. Both single-layer orthotropic and two-layer cross-ply laminated plate constructions are considered.
3. Thickness shear deformations are included.
4. Temperature changes through the thickness as well as in the plane are considered.
5. Both finite-element and exact closed-form solutions are presented.

The objective of the present paper is two-fold: to review the recent developments in the analyses of bimodulus-material plates with finite dimensions and to present new finite-element results for rectangular plates under conditions that do not admit closed-form solutions.

2. Governing Equations for Bimodulus Plates

In the following the equations governing the shear deformable theory of layered composite plates (see Whitney and Pagano [24]) is reviewed and the thermoelastic constitutive equations of bimodulus-material plates are presented. In deriving the governing equations it is assumed that all of the layers in the plate remain elastic during the deformation, the generalized Hooke's law is valid, and no slip occurs between any two layers.

Consider a plate constructed of a finite number of uniform thickness, orthotropic, bimodulus layers with arbitrary orientations (i.e., the material symmetry axes of each layer do not coincide, in general with the plate axes). The x-y plane lies in the middle plane, R, of the plate and the z-axis is normal to R. The displacement field in the shear deformable theory of plates is assumed to be given by

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\psi_x(x,y,t) \\
v(x,y,z,t) &= v_0(x,y,t) + z\psi_y(x,y,t) \qquad (2.1) \\
w &= w(x,y,t)
\end{aligned}
$$

where u, v, and w are the displacements along x, y and z directions, respectively, u_0 and v_0 are the in-plane displacements of the middle plane, ψ_x and ψ_y are the bending slopes, and t is time. Using the small-deflection theory, the strain-displacement relations can be expressed in the form

$$
\begin{aligned}
&\varepsilon_1 \equiv \varepsilon_x = u_{o,x} + z\psi_{x,x} \;, \quad \varepsilon_2 \equiv \varepsilon_y = v_{o,y} + z\psi_{y,y}, \\
&\varepsilon_6 \equiv \gamma_{xy} = u_{o,y} + v_{o,x} + z(\psi_{x,y} + \psi_{y,x}) \;, \qquad (2.2) \\
&\gamma_{xz} = \psi_x + w,_x \;, \quad \gamma_{yz} = \psi_y + w,_y \;, \quad \varepsilon_3 = 0
\end{aligned}
$$

where $w,_x \equiv \partial w/\partial x$, etc.

The equations of motion are given by,

$$
\begin{aligned}
N_{1,x} + N_{6,y} &= Pu_{o,tt} + R\psi_{x,tt} \\
N_{6,x} + N_{2,y} &= Pv_{o,tt} + R\psi_{y,tt} \\
Q_{1,x} + Q_{2,y} &= q + Pw_{,tt} \qquad (2.3) \\
M_{1,x} + M_{6,y} - Q_1 &= I\psi_{x,tt} + R\,u_{o,tt} \\
M_{6,x} + M_{2,y} - Q_2 &= I\psi_{y,tt} + R\,v_{o,tt}
\end{aligned}
$$

where P, R, and I are the normal, coupled-normal-rotatory, and rotatory inertia coefficients,

$$
(P,R,I) = \sum_m \int_{z_m}^{z_{m+1}} (1,z,z^2)\rho^{(m)}\, dz \qquad (2.4)
$$

$\rho^{(m)}$ being the material density of the m-th layer, q is the transversely distributed force, and N_i, Q_i, and M_i are the respective inplane and transverse stress and moment resultants defined by

$$(N_i, M_i) = \int_{-h/2}^{h/2} (1,z)\sigma_i \, dz \quad (i = 1,2,6) \quad , \quad (Q_1, Q_2) = \int_{-h/2}^{h/2} (\sigma_5, \sigma_4) dz, \tag{2.5}$$

where h is the plate (laminate) thickness, and the so-called contracted subscript notation is employed to denote the stress components.

Assuming that the only plane of symmetry existing is in the plane of the plate, the thermoelastic constitutive relations for each layer (ℓ) are taken to be monoclinic and bimodular as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11k\ell} & Q_{12k\ell} & 0 \\ Q_{12k\ell} & Q_{22k\ell} & 0 \\ 0 & 0 & Q_{66k\ell} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 - \alpha_{1k\ell}T \\ \varepsilon_2 - \alpha_{2k\ell}T \\ \varepsilon_6 \end{Bmatrix} \tag{2.6}$$

$$\begin{Bmatrix} \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} C_{44k\ell} & C_{45k\ell} \\ C_{45k\ell} & C_{55k\ell} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \tag{2.7}$$

Here T is the temperature change measured from the strain-free temperature, the C's are Cauchy elastic shear stiffnesses, the Q's are plane-stress reduced stiffnesses, and the α's are thermal-expansion coefficients. The third subscript in $Q_{ijk\ell}$ and $C_{ijk\ell}$ (and second in $\alpha_{jk\ell}$) refers to the bimodular characteristics: k = 1 denotes properties associated with fiber-direction tension, k = 2 denotes fiber-direction compression. Also, subscript ℓ refers to the individual layer number, i.e., ℓ = 1 and 2 for a two-layer laminate.

Substituting (2.2), (2.6) and (2.7) into (2.5), we obtain the constitutive equations for an arbitrarily laminated plate,

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \left[\begin{array}{c|c} [A] & [B] \\ \hline [B]^T & [D] \end{array}\right] \begin{Bmatrix} \{\varepsilon^\circ\} \\ \{\kappa\} \end{Bmatrix} - \begin{Bmatrix} \{N^T\} \\ \{M^T\} \end{Bmatrix} , \tag{2.8}$$

$$\begin{Bmatrix} Q_2 \\ Q_1 \end{Bmatrix} = \begin{bmatrix} S_{44} & S_{45} \\ S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} , \tag{2.9}$$

where

$$\begin{aligned} \varepsilon_1^0 &= u_{0,x} \ , \quad \varepsilon_2^0 = v_{0,y} \ , \quad \varepsilon_6^0 = u_{0,y} + v_{0,x} \\ \kappa_1 &= \psi_{x,x} \ , \quad \kappa_2 = \psi_{y,y} \ , \quad \kappa_6 = \psi_{x,y} + \psi_{y,x} \end{aligned} \tag{2.10}$$

and A_{ij}, B_{ij}, D_{ij}, S_{ij} are the respective inplane, bending-inplane coupling, bending or twisting, and thickness-shear stiffnesses defined as follows:

$$\begin{aligned} (A_{ij},B_{ij},D_{ij}) &= \int_{-h/2}^{h/2} (1,z,z^2) Q_{ijk\ell}\, dz \quad (i,j = 1,2,6) \\ S_{ij} &= K_i K_j \int_{-h/2}^{h/2} Q_{ijk\ell}\, dz \quad (i,j = 4,5) \end{aligned} \tag{2.11}$$

The stress and moment resultants, N_i^T and M_i^T, due to thermal loading, are defined by

$$(N_i^T, M_i^T) = \sum_m \int_{z_m}^{z_{m+1}} \sum_j^2 Q_{ijk\ell}\, \alpha_{jk\ell}(T_0, zT_1) dz \quad , \quad (i,j = 1,2,6) \tag{2.12}$$

where the temperature change from a reference state is assumed to be linear through the thickness, consistent with the plate theory:

$$T(x,t,z) = T_0(x,y) + zT_1(x,y). \tag{2.13}$$

This completes the description of the field equations. Closed-form solutions to these equations are possible in the particular case of a cross-ply plate whose stiffness coefficients (A_{ij}, B_{ij} and D_{ij}) with subscripts 16, 26, and 45 are zero, and whose edges are hinged flexurally but free to move in a direction normal to each edge. The temperature distribution and the normal-pressure loadings must be sinusoidal in order to obtain the closed-form solution (see [18-23]).

3. Finite-Element Formulation

The finite-element formulation of the equations presented in the previous section follows along the same lines as for ordinary (i.e., not bimodulus) plates (see Reddy [25]). The main difference lies in the computation of the stiffness coefficients, which depend on the stresses in the plate. We summarize the results in the following.

Consider a finite-element representation of the midplane of the plate by quadrilateral isoparametric elements. Over a typical element, R^e, the displacements (u_o, v_o, w, ψ_x, ψ_y) are approximated by expressions

$$u_o = \sum_{i=1}^{n} u_i\phi_i \quad , \quad v_o = \sum_{i}^{n} v_i\phi_i, \text{ etc.} \tag{3.1}$$

where $\phi_i(x,y)$ $(i = 1,2,\ldots,n)$ are interpolation functions associated with the linear $(n = 4)$ or quadratic $(n = 8$ or $9)$ rectangular element. Substituting (3.1) into the weak form associated with (2.3), (2.8), and (2.9), we obtain

$$([K] - \omega^2[M])\{\Delta\} = \{F\} \tag{3.2}$$

The elements of the stiffness matrix [K] and mass matrix [M] are given by

$$\begin{aligned}
&K^{11}_{ij} = A_{11}G^{x}_{ij} + A_{66}G^{y}_{ij} \quad , \quad K^{12}_{ij} = A_{12}G^{xy}_{ij} + A_{66}G^{xy}_{ji} \ , \\
&K^{14}_{ij} = B_{11}G^{x}_{ij} + B_{66}G^{y}_{ij} \quad , \quad K^{15}_{ij} = B_{12}G^{xy}_{ij} + B_{66}G^{xy}_{ji} \ , \\
&K^{22}_{ij} = A_{22}G^{y}_{ij} + A_{66}G^{x}_{ij} \quad , \quad K^{24}_{ij} = B_{66}G^{xy}_{ij} + B_{12}G^{xy}_{ji} \ , \\
&K^{25}_{ij} = B_{66}G^{x}_{ij} + B_{22}G^{y}_{ij} \quad , \quad K^{33}_{ij} = S_{55}G^{x}_{ij} + S_{44}G^{y}_{ij} \ , \\
&K^{34}_{ij} = S_{55}G^{x}_{ij} + S_{44}G^{y}_{ij} \quad , \quad K^{35}_{ij} = S_{44}G^{yo}_{ij} \ , \\
&K^{44}_{ij} = D_{11}G^{x}_{ij} + D_{66}G^{y}_{ij} + S_{55}G^{o}_{ij} \quad , \quad K^{45}_{ij} = D_{12}G^{xy}_{ij} + D_{66}G^{xy}_{ji} \ , \\
&K^{55}_{ij} = D_{66}G^{x}_{ij} + D_{22}G^{y}_{ij} + S_{44}G^{o}_{ij} \quad , \quad K^{13}_{ij} = K^{23}_{ij} = 0 \ ,
\end{aligned} \tag{3.3}$$

$$M_{ij}^{\alpha\alpha} = PG_{ij}^{00} \quad , \quad (\alpha = 1,2,3) \; ,$$
$$M_{ij}^{\alpha\beta} = RG_{ij}^{00} \quad , \quad (\alpha,\beta = 4,5;\ \alpha \neq \beta) \; ,$$
$$M_{ij}^{\alpha\alpha} = IG_{ij}^{00} \quad , \quad (\alpha = 4,5) \; , \qquad (3.4)$$

$$F_i^{\alpha} = \int_{R^e} f_\alpha \, \phi_i \, dx\, dy \quad , \quad (\alpha = 1,2,3,4,5) \qquad (3.5)$$

$$G_{ij}^{\xi\eta} = \int_{R^e} \phi_{i,\xi} \, \phi_{j,\eta} \, dx\, dy \quad , \quad (\xi,\eta = 0,1,2). \qquad (3.6)$$

$$f_1 = N_{1,x}^T + N_{6,y}^T \quad , \quad f_2 = N_{6,x}^T + N_{2,y}^T \quad , \quad f_3 = q \quad ,$$
$$f_4 = M_{1,x}^T + M_{6,y}^T \quad , \quad f_5 = M_{6,x}^T + M_{2,y}^T \; . \qquad (3.7)$$

In the present study the 4-node and 8-node rectangular isoparametric elements are employed.

4. Numerical Results and Discussion

Numerical results for deflections, neutral-surface locations, and vibration frequencies are presented for plates under various edge conditions and loadings. The discussion of the results is divided into three parts according to bending due to normal pressure, bending due to thermal loading, and free vibration. In all three parts, numerical results are presented first for plate problems which admit closed-form solutions. All of the finite-element results presented herein were obtained on an IBM 370/158 computer using the double-precision arithmetic.

Table 1. Elastic properties for two tirecord-rubber, unidirectional, bimodulus composite materials

Property and Units	Aramid Rubber (AR) Tension (k=1)	Aramid Rubber (AR) Compression (k=2)	Polyester-Rubber (PR) Tension (k=1)	Polyester-Rubber (PR) Compression (k=2)
Longitudinal Young's modulus, GPa (E_{11})	3.58	0.0120	0.617	0.0369
Transverse Young's modulus, GPa (E_{22})	0.00909	0.0120	0.00800	0.0106
Major Poisson's ratio, dimensionless[a] (ν_{12})	0.416	0.205	0.475	0.185
Longitudinal-transverse shear modulus, GPa[b] (G_{12})	0.00370	0.00370	0.00262	0.00267
Transverse-thickness shear modulus, GPa (G_{23})	0.00290	0.00499	0.00233	0.00475

[a]The minor Poisson's ratio is assumed to be given by the reciprocal relation.
[b]The longitudinal-thickness shear modulus is assumed to be equal to the longitudinal-transverse shear modulus.

Static Bending Due to Normal Pressure

To show the accuracy of the finite-element solutions, a comparison is made of the present results with the closed-form results obtained in [21]. Table 2 contains the neutral-surface locations, and the nondimensionalized transverse deflection of single-layer and two-layer (0°/90°), simply-supported (SS), rectangular plates (aramid-rubber) under sinusoidally (SSL) distributed transverse load, $q = q_0 \sin(\pi x/a)\sin(\pi y/b)$. As can be seen from Table 2, the agreement between the closed-form solution (CFS) and finite-element solution (FES) is extremely good. It should be noted that if the material is treated as an ordinary material, with material properties taken as the average of compressive and tensile properties, the deflections will be under-estimated significantly.

Table 2. Neutral-surface locations and dimensionless center deflections for simply-supported thick rectangular plates of aramid-rubber (b/h = 10, shear correction factor = 5/6) subjected to sinusoidally distributed normal-pressure loading

b/a	Single-layer					Two-layer (0°/90°)[2]				
	$Z_x = z_{nx}/h$		$\bar{w} = (w_0E^C_{22}h^3/q_0b^4)10^2$			$Z_x = z_{nx}/h$		$\bar{w} = (w_0E^C_{22}h^3/q_0b^4)10^2$		
	CFS	FES	CFS	FES	Average[1] (FES)	CFS	FES	CFS	FES	Average[1] (FES)
0.6	0.4452	0.4452	0.456	0.456	0.1359	0.4427	0.4426	0.4388	0.434	0.2186
0.8	0.4440	0.4440	1.105	1.106	0.2546	0.4407	0.4407	1.054	1.047	0.3782
1.0	0.4420	0.4420	2.046	2.052	0.4239	0.4384	0.4384	1.957	1.946	0.5453
1.2	0.4394	0.4394	3.160	3.172	0.6561	0.4356	0.4357	3.043	3.029	0.6926
1.4	0.4363	0.4363	4.313	4.331	0.9664	0.4326	0.4326	4.185	4.167	0.8078
1.6	0.4328	0.4329	5.406	5.434	1.3724	0.4292	0.4293	5.282	5.261	0.8918
1.8	0.4292	0.4294	6.390	6.426	1.8943	0.4257	0.4257	6.277	6.254	0.9511
2.0	0.4253	0.4254	7.250	7.293	-	0.4219	0.4219	7.151	7.127	-

[1]Results obtained by using the average of the compressive and tensile properties listed in Table 1.

[2]Neutral-surface location Z_y is not listed here (see [9]).

The aramid-rubber plates, in both the single- and cross-ply cases, have noticeably larger values of Z_x than the polyester-rubber ones, due to the more pronounced bimodulus effect in the fiber-direction Young's modulus of the aramid-rubber. There are only very slight differences in Z_x and deflection in going from a single-layer plate to a cross-ply laminate. This is in considerable contrast to ordinary materials.

Having validated the finite-element model, the model is then used for other combinations of loading and boundary conditions that are not amenable to closed-form solutions. Figure 1 shows plots of the nondimensionalized deflection versus the side-to-thickness ratio for two-layer (0°/90°) simply-supported and clamped (CC) square plates under various loadings. The effect of the thickness shear on the deflections is apparent from the graphs.

Static Bending Due to Thermal Loading

Next, numerical results are presented for static bending of plates under thermal gradients only (i.e., $\bar{T}_0 = 0$). The same orthotropic bimodulus materials as listed in Table 1 are used. Since no measured values of thermal-expansion coefficients of these materials are available, the following ratios of the thermal-expansion coefficients are used:

$$\alpha_1^t/\alpha_1^c = 0.5 \quad ; \quad \alpha_2^t/\alpha_2^c = 1.0 \quad ; \quad \alpha_1^t/\alpha_2^t = 0.1. \qquad (4.3)$$

To show the accuracy of the finite-element solutions, a comparison is made of the present results with the closed-form results obtained in [22]. Table 3 contains the neutral-surface locations, and the

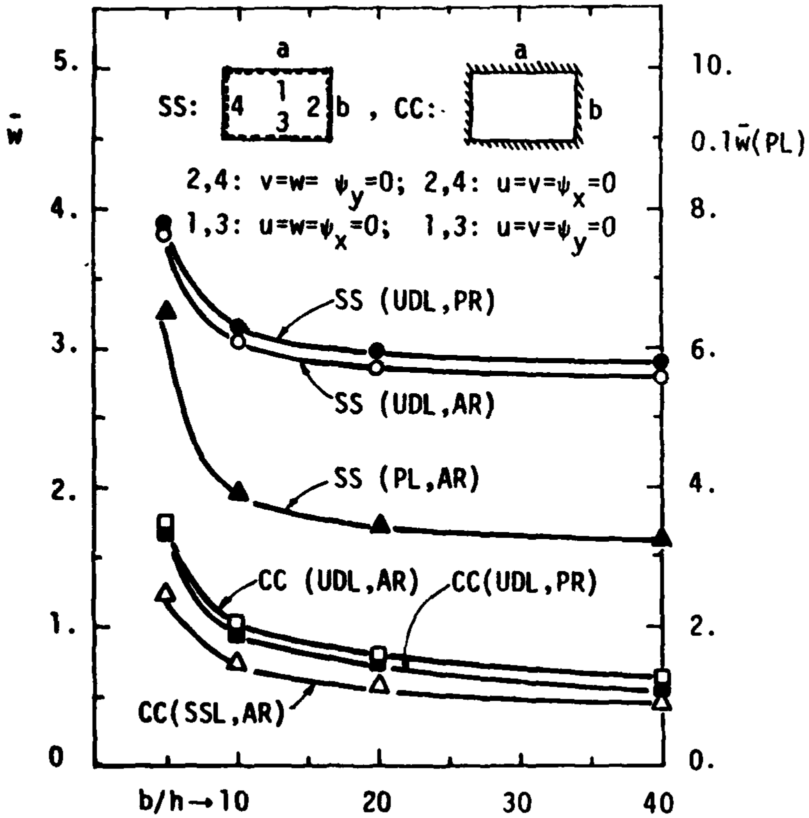


Figure 1 Nondimensionalized center deflection vs. side-to-thickness ratio for two-layer simply supported (SS) and clamped (CC) square plates under uniformly distributed load (UDL) point load (PL) at the center.

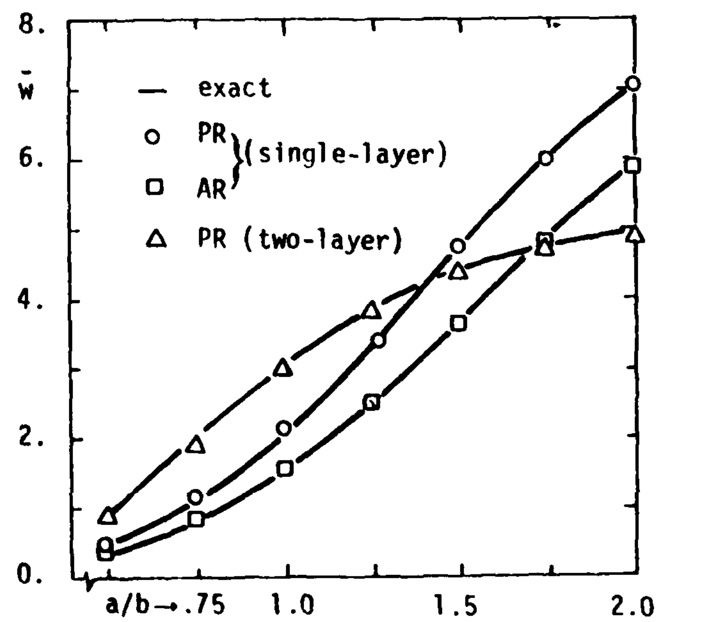


Figure 2 Nondimensionalized center deflection vs. aspect ratio for simply supported rectangular plates (b/h = 10) under sinusoidal temperature gradient through the thickness.

nondimensionalized transverse deflection of single-layer and two-layer (0°/90°), simply-supported (SS), rectangular plates (polyester-rubber) under sinusoidally (SST) distributed temperature, $T = \bar{T}_1 a \sin(\pi x/a) \sin(\pi y/b)$. As can be seen from Table 3, the agreement between the closed-form solution (CFS) and finite-element solution (FES) is extremely good (see also Fig. 2).

Table 3. Neutral-surface locations and dimensionless center deflections for simply-supported thick ($b/h = 10$, $K_1 = K_2 = 5/6$) rectangular plates of polyester-rubber subjected to sinusoidal temperature gradient, $T = z\bar{T}_1 \sin(\pi x/a)\sin(\pi y/b)$.

a/b	single-layer				two-layer +			
	Z_x (= z_x/h)		$\bar{w}^{++}$		Z_x (= z_x/h)		$\bar{w}$	
	CFS	FES	CFS	FES	CFS	FES	CFS	FES
0.50	0.1031	0.1030	0.482	0.482	0.2599	0.2541	0.928	0.894
0.75	0.1184	0.1183	1.157	0.158	0.2554	0.2436	0.199	0.187
1.00	0.1308	0.1308	2.160	2.161	0.2398	0.2294	0.309	0.296
1.25	0.1360	0.1360	3.410	3.409	0.2119	0.2035	0.392	0.382
1.50	0.1332	0.1331	4.737	4.736	0.1734	0.1679	0.443	0.436
1.75	0.1234	0.1233	5.975	5.970	0.1284	0.1252	0.472	0.466
2.0	0.1078	0.1078	7.024	7.017	0.0815	0.0804	0.489	0.483

+Neutral-surface location Z_y is not listed here (see [22])

++$\bar{w} = (wh/\alpha_1^t\, T_1\, b^2)10$

Next, finite-element results are presented for other combinations of loading and boundary conditions that are not amenable to closed-form solutions. Figure 3 shows plots of the nondimensionalized deflection versus the side-to-thickness ratio for two-layer (0°/90°) and one-layer simply-supported (SS) square plates (polyester-rubber) under uniform temperature gradient through the thickness. The effect of the thickness shear on the deflections is apparent from the graphs.

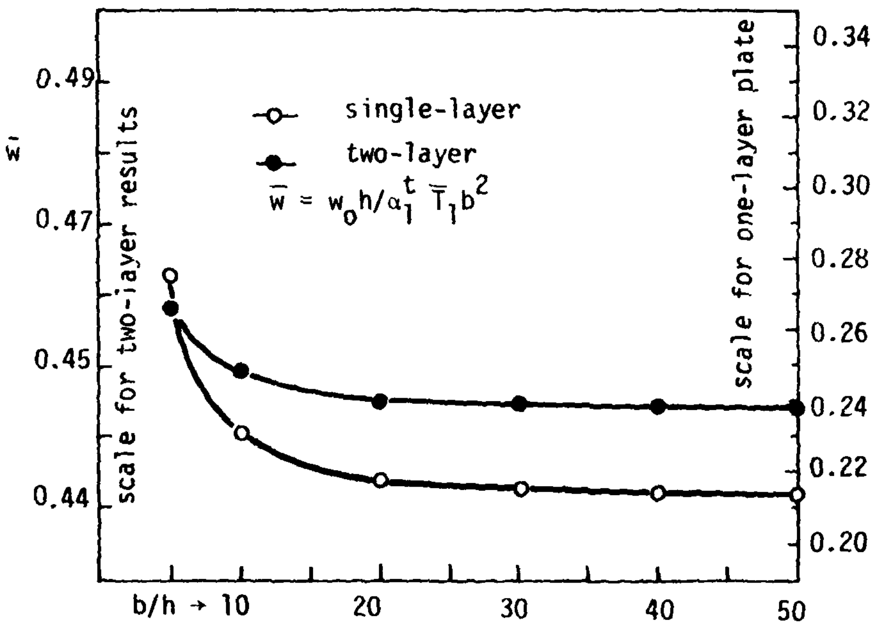


Fig.3 Nondimensionalized center deflection vs. side-to-thickness ratio for two-layer simply supported (SS) square plates under uniformly distributed temperature gradient through the thickness.

Natural Vibration Results

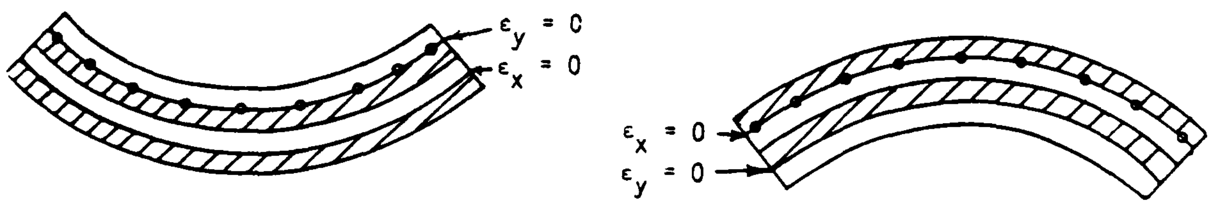
Before we present the numerical results of free vibration, we discuss the effect of bimodulus action on plate stiffness in different portions of a vibration cycle. First consider a single-layer, bimodulus material plate at the two extremes of its deflection (Fig. 4). During the first half cycle (see Fig. 4a) the top surface is in compression and the bottom in tension, thus causing the neutral surface for ε_x to be positive ($z_{nx} > 0$), i.e., a certain distance below the plate midplane. During the second half cycle (Fig. 4b) the top surface is in tension and the bottom is in compression, just opposite to that encountered in the first half cycle. However, the magnitude of z_{nx} is the same as in the first half cycle. Thus, it can be concluded that the effective stiffness (and thus the frequency) associated with the second half cycle is identical to that of the first half cycle and either modal shape, Fig. 4a or 4b, will give the same computational result for the natural frequencies.

Now consider a two-ply laminate with the bottom layer (layer $\ell = 1$) oriented at 0° and the top layer ($\ell = 2$) at 90° (see Fig. 4). Initially, as shown in Fig. 4c, the neutral surface for ε_x falls below the interface, within the 0° layer, while the neutral surface for ε_y falls above the interface, completely within the 90° layer. In the latter portion of the cycle, Fig. 4d, the ε_x neutral surface falls outside of the layer, and the ε_y neutral surface falls outside of the 90° layer. Therefore, compressive properties are used for the entire 0° layer, and tensile ones for the 90° layer. Thus, for a two-layer cross-ply laminate the plate stiffnesses acting in the two portions of a cycle are different and the associated frequencies are also different (except for a square

(A) FIRST HALF CYCLE

(B) SECOND HALF CYCLE

SINGLE-LAYER, 0° BIMODULUS PLATE



(C) FIRST PORTION OF CYCLE

(D) SECOND PORTION OF CYCLE

TWO-LAYER, CROSS-PLY PLATE
(BOTTOM LAYER AT 0°, TOP LAYER AT 90°)

Fig. 4 Bimodulus action in single-layer and cross-ply plates in the fundamental mode of vibration. Cross-hatched regions denote tension in the respective fiber directions.

plate). Denoting the frequencies associated with the two portions of a vibration cycle by ω_1 and ω_2, we can easily show that the average frequency (ω) over the entire cycle is

$$\omega^{-1} = (1/2)(\omega_1^{-1} + \omega_2^{-1}) \tag{4.4}$$

Thus, the computational procedure required for a cross-ply plate is to calculate ω_1 and ω_2 associated with modal shapes shown in Figs. 4c and 4d, respectively, and then to apply equation (4.4).

The question arises as to the possibility of a discontinuity in energy at the instant of time between the two portions of a cycle. However, it was shown in [23] that there is no such discontinuity.

Numerical results are presented for relatively thick plates (b/h = 10) of aramid-rubber. Plate aspect ratio affects the fundamental frequency much more than it does the neutral-surface portion (see Tables 4-5). For the cross-ply case, only $Z_x^{(1)}$ and $Z_x^{(2)}$ is close to $Z_y^{(1)}$. In Fig. 5 the good agreement between the CFS and FES results is clearly shown.

Since there are very drastic changes in neutral-surface locations (for example, z_{nx} for aramid-rubber goes from approximately 0.4h to -0.03h) from one cycle portion to the other, there is a question that may arise regarding a possible transient action. However, the neutral surfaces are just boundaries between the tensile and compressive regions (analogous to the elastic-plastic boundary in elastoplastic problems). Thus, they have no mass and there is no transient action.

Table 4 Dimensionless fiber-direction neutral-surface locations for single-layer (0°) and two-layer (0°/90°) plates of polyester-rubber material ($b/h = 10$).

	single-layer (0°); polyester-rubber				two-layer (0°/90°); polyester-rubber[1]							
a/b	$\omega_0 b^2(P/E_{22}^C h^3)^{1/2}$		$Z_x = z_{nx}/h$		$Z_x^{(1)}$		$-Z_y^{(1)}$		$-Z_x^{(2)}$		$Z_y^{(2)}$	
	CFS	FES	CFS	FES	CFS	FES	CFS	FES	CFS	FES	CFS	FES
0.5	25.134	23.136	0.3089	0.3083	0.3687	0.3691	0.1335	0.1295	0.0830	0.0825	0.3569	0.357
0.7	15.058	14.421	0.3072	0.3071	0.3664	0.3663	0.1119	0.1113	0.0868	0.0868	0.3603	0.360
1.0	8.668	8.648	0.3056	0.3049	0.3632	0.3633	0.0960	0.0960	0.0959	0.0959	0.3631	0.363
1.4	5.421	5.533	0.3011	0.3013	0.3589	0.3596	0.0870	0.0870	0.1115	0.112	0.3648	0.365
2.0	3.777	3.918	0.2945	0.2950	0.3514	0.3513	0.0817	0.0817	0.1389	0.139	0.3660	0.366

[1] $Z_x^{(i)}$ denotes z_{nx}/h for the i-th ($i = 1,2$) portion of the cycle.

Table 5 Dimensionless fundamental frequencies in the first partial cycle, second partial cycle, and complete cycle of motion for two-layer, cross-ply plates having b/h = 10 by closed form and finite element methods.

	$\omega_1 b^2(P/E^c_{22}h^3)^{1/2}$		$\omega_2 b^2(P/E^c_{22}h^3)^{1/2}$		$\omega b^2(P/E^c_{22}h^3)^{1/2}$	
a/b	CFS	FES	CFS	FES	CFS	FES
			Aramid-Rubber:			
0.5	19.38	20.23	13.88	14.55	16.18	16.93
0.7	11.60	12.17	9.353	9.807	10.35	10.86
1.0	7.038	7.386	7.038	7.364	7.038	7.375
1.4	4.838	5.045	6.037	6.356	5.371	5.625
2.0	3.712	3.909	5.551	5.821	4.449	4.677
			Polyester-Rubber:			
0.5	19.12	19.81	15.95	16.61	17.39	18.07
0.7	11.43	11.92	10.04	10.45	10.69	11.14
1.0	7.084	7.406	7.085	7.394	7.085	7.400
1.4	5.164	5.407	5.928	6.193	5.520	5.773
2.0	4.310	4.518	5.435	5.688	4.807	5.036

5. Summary and Conclusions

Both closed-form and finite-element solutions are presented for static bending and free vibrations of single-layer and two-layer cross-ply plates of bimodulus composite materials. In the case of simply supported plates under sinusoidal temperature distribution, normal-pressure loading, and free vibration, the closed-form and the finite-element solutions for neutral-surface locations, center deflections, and fundamental frequencies are in excellent agreement. The finite-element method is then used to obtain solutions for simply-supported plates under uniform loading, for which no closed-form solution can be obtained for the bimodulus case.

The research reported here has recently been extended to mechanically loaded thin cylindrical shells [26], free vibration of thin and thick cylindrical shells [27], and thermally and mechanically loaded thick cylindrical shells [28].

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18. SUPPLEMENTARY NOTES

A portion of this paper was presented as "Behavior of Plates Laminated of Bimodulus Materials" at the First Japan-U.S. Conference on Composite Materials, Tokyo, Japan, Jan. 12-14, 1981, and will appear in its Proceedings. The thermal bending portion of this paper forms part of "On Thermal Bending (over)

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Bimodulus materials, composite materials, fiber-reinforced materials, finite-element method, free vibration, laminates, natural frequencies, plate bending, thermal stresses, vibration.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An exact and finite-element analysis is used to predict the static bending and natural vibration response of rectangular plates laminated of composite material having different elastic properties depending upon whether the fiber-direction strains are tensile or compressive. The analysis is based on a theory that accounts for anisotropy, bending-stretching coupling, thickness shear deformation, and both coupling inertia and rotatory inertia. The finite-element results are found to agree very closely with the exact closed-form solutions in the case of a cross-ply rectangular plate having simply (over)

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20. Abstract - Cont'd

supported edges and subjected to sinusoidally distributed normal-pressure loading or temperature loading.

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